The strong coupling constant $g_{D^*D\pi}$ and final-state interactions

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Abstract

In this article, we study the contribution from the final-state interactions to the strong coupling constant $g_{D^*D\pi}$. We take an assumption that the momentum transfers in the strong decay $D^{*+} \to D^0\pi^+$ be large to validate the operator product expansion in the light-cone QCD sum rules. At large momentum transfers, the final-state interactions play an important role, and we should take them into account.

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1 Introduction

Several QCD sum rules approaches have been applied to determine the strong coupling constant $g_{D^*D\pi}$ in the strong decay $D^{*+} \to D^0\pi^+$, such as two-point correlation function with soft-pion technique [1, 2], or beyond the soft-pion approximation [3], light-cone QCD sum rules [4, 5], light-cone sum rules with perturbative α_s corrections [6], QCD sum rules in a external field [7], double-moment QCD sum rules [8], and double Borel sum rules [9], etc. The discrepancy between the experimental data from the CLEO collaboration and the predictions from the QCD sum rules is very large. The upper bound $g_{D^*D\pi} = 13.5$ ($g_{D^*D\pi} = 10.5 \pm 3.0$ from the light-cone QCD sum rules with perturbative α_s corrections to the twist-2 light-cone distribution amplitude $\phi_\pi(\mu, u)$ [6]) is too small to account for the experimental data, $g_{D^*D\pi} = 17.9 \pm 0.3 \pm 1.9$ [10].

It has been noted that the simple quark-hadron duality ansatz which works in the one-variable dispersion relation might be too crude for the double dispersion relation [11]. In Ref.[12], the authors observe that inclusion of the contributions from an explicit radial excitation to the hadronic spectral density can improve the value of $g_{D^*D\pi}$ significantly, however, additional (strong) assumptions about the strong coupling constants concerning the radial excitations are taken. On the other hand, in Ref.[9], the authors find that in the standard QCD sum rules, a modification of the contribution from the continuum states may lead to unstable sum rules. In Ref.[13], the authors argue that the subtracting term $M^2 e^{-\frac{s_0}{M^2}}$ comes from a mathematically

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spurious term and it should not be a part of the final sum rules, however, absence of the continuum states subtraction seems rather strange.

In Ref.[14], the form-factor $g_{D^*D\pi}(Q^2)$ for off-shell D meson is evaluated at low and moderate Q^2 in a hadronic loop model. The authors fix the arbitrary constants to match previous QCD sum rule calculations valid at higher Q^2, then extrapolate to the mass shell to obtain the coupling constant $g_{D^*D\pi}$.

Despite large uncertainties, the QCD sum rules have given a great deal of good agreements with the experiment data. The strong coupling constant $g_{D^*D\pi}$ seems to be exotic. In the heavy quark limit, a quark model based on the Dirac equation in a central potential leads to the value $g_{D^*D\pi} \approx 18$ [15]. The quenched lattice QCD calculation results in $g_{D^*D\pi} = 18.8 \pm 2.3^{+1.1}_{-2.0}$ [16].

We study the strong coupling constant $g_{D^*D\pi}$ with the two-point correlation function $\Pi_\mu(p,q)$ [4, 6],

$$\Pi_\mu(p,q) = i\int d^4x\, e^{-iq\cdot x} \langle 0|T\left\{J_\mu(0)J_5^+(x)\right\}|\pi(p)\rangle\,, \tag{1}$$

$$J_\mu(x) = \bar{u}(x)\gamma_\mu c(x)\,,$$

$$J_5(x) = \bar{d}(x)i\gamma_5 c(x)\,, \tag{2}$$

where the currents $J_\mu(x)$ and $J_5(x)$ interpolate the mesons D^* and D, respectively. The external state π has the four momentum p_μ with $p^2 = m_\pi^2$. In this article, we take the isospin limit for the u and d quarks.

The calculations are performed at large spacelike momentum regions $(q+p)^2 \ll 0$ and $q^2 \ll 0$, which correspond to small light-cone distance $x^2 \approx 0$ required by validity of the operator product expansion [17, 18]. In the strong decay $D^{*+} \to D^0\pi^+$, the momentum transfers $D^* \to \pi$ and $D^* \to D$ are very small, $m_{D^*} \approx m_D + m_\pi = (1.87+0.14)\text{GeV}$. In the light-cone QCD sum rules, we perform the operator product expansion at large momentum transfers, at that energy scale, the final-states are active, their interactions may play an important role and we should take them into account. In this article, we study the final-state interactions (elastic scatterings) of $D\pi$ with Bethe-Salpeter re-summation.

Take the amplitudes from the chiral Lagrangian as kernels, and solve the corresponding Bethe-Salpeter equation, we can re-sum an infinite series of loop diagrams in chiral expansions, generate quasi-bound states of the mesons (or baryons) dynamically, and account for the resonances without including them explicitly [19].

The article is arranged as: in Section 2, we perform Bethe-Salpeter re-summation for the final-state interactions in the strong decay $D^{*+} \to D^0\pi^+$; in Section 3, the numerical result and discussion; and in Section 4, conclusion.

2 Bethe-Salpeter re-summation for the final-state interactions

In order to describe the interactions between the light and heavy pseudoscalar mesons, we employ the leading order heavy chiral Lagrangian [20],

$$\mathcal{L} = \frac{1}{4f_\pi^2}\left\{\partial^\mu P[\Phi, \partial_\mu \Phi]P^\dagger - P[\Phi, \partial_\mu \Phi]\partial^\mu P^\dagger\right\}, \tag{3}$$

where $f_\pi = 92.4$MeV is the weak decay constant of the π, P stand for the charmed mesons D^0, D^+ and D_s^+, and Φ denote the octet pseudoscalar mesons,

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}. \tag{4}$$

The amplitude V for the elastic scattering $D^+\pi^- \to D^+\pi^-$ can be obtained from the leading order heavy chiral Lagrangian,

$$V_{D^+\pi^-}(s, t, u) = \frac{s-u}{4f_\pi^2}, \tag{5}$$

where the s, t and u are Mandelstam variables [2],

$$\begin{aligned} -u(s, \cos\theta) &= s - m_\pi^2 - m_D^2 - 2\left(m_D^2 + \frac{\lambda^2(s, m_\pi^2, m_D^2)}{4s}\right) + \frac{\lambda^2(s, m_\pi^2, m_D^2)}{2s}\cos\theta, \\ \lambda(s, m_D^2, m_\pi^2) &= \sqrt{[s-(m_D+m_\pi)^2][s-(m_D-m_\pi)^2]}. \end{aligned} \tag{6}$$

In unitary chiral perturbation theory, with on-shell approximation, the full scattering amplitude T can be converted into an algebraic Bethe-Salpeter equation [19],

$$T = (1 - VG)^{-1}V = V + VGV + VGVGV + \cdots, \tag{7}$$

where $V = V_{D^+\pi^-}(s, t, u)$ and

$$G(p^2) = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2 - m_D^2 + i\epsilon}\frac{1}{(p-q)^2 - m_\pi^2 + i\epsilon}, \tag{8}$$

$$\mathrm{Re}G(s) = \frac{1}{4\pi^2}\mathrm{P}\int_0^{q_{max}} dq \frac{q^2(\omega_D + \omega_\pi)}{\omega_D\omega_\pi[s - (\omega_D + \omega_\pi)^2]}, \tag{9}$$

$$\mathrm{Im}G(s) = -\frac{\lambda(s, m_D^2, m_\pi^2)}{16\pi s}, \tag{10}$$

here $\mathrm{P}\int$ stands for the principal integral, $\omega_i = \sqrt{q^2 + m_i^2}$, $q = |\overrightarrow{q}|$ in Eq.(9).

[2] For technical details, one can consult the Ph.D thesis (in Chinese) of F. K. Guo, Institute of high energy physics.

Taking into account the final-state interactions of the D and π, the strong coupling constant takes the following form,

$$g_{D^*D\pi} \to g g_{D^*D\pi} = g_{D^*D\pi} - g_{D^*D\pi} G(s) T_1(s) \,, \tag{11}$$

here we introduce g to denote the enhanced form-factor comes from the final-state interactions.

$$T(s,t,u) = \sum_{l=0}^{\infty} (2l+1) L_l(cos\theta) T_l(s) \,, \tag{12}$$

where $L_l(cos\theta)$ are Legendre polynomials, the strong decay $D^{*+} \to D^0\pi^+$ takes place through relative P-wave, we take the $l=1$ partial wave amplitude $T_1(s)$.

3 Numerical result and discussion

There is a singular point at

$$s - (\sqrt{q^2 + m_D^2} + \sqrt{q^2 + m_\pi^2})^2 = 0 \,, \tag{13}$$

in the principal integral. In this article, we take the value of the q_{max} be the typical energy scale $q_{max} = m_\rho = 0.77\text{GeV}$ in the chiral perturbation theory, the value of s should be $s > 7.9\text{GeV}^2$ to avoid the singular point. If we take s be the center of mass of the vector meson D^*, $s = m_{D^*}^2$, then $q_{max} \approx 0$, the final-state interactions are of minor importance and can be neglected safely.

In Fig.1, we plot the enhanced factor $|g|$ with the variation of the center of mass parameter s. From the figure, we can see the value of $|g|$ increases quickly according to s.

We take the momentum transfer $D^* \to \pi$ in the strong decay $D^{*+} \to D^0\pi^+$ be large to validate the operator product expansion in the light-cone QCD sum rules. The deviation $s - m_{D^*}^2$ measures the virtuality of the initial vector meson D^*, we introduce the effective mass m_{eff} to denote the virtual mass of the vector meson D^*, where the momentum transfer in $D^* \to \pi$ is large enough. If we choose the typical value $s = m_{eff}^2 = (m_{D^*} + m_D)^2$, the enhanced factor $|g|$ is rather large, $|g| \approx 1.4$. We can take the value from the light-cone QCD sum rules with perturbative α_s corrections as input parameter, $g_{D^*D\pi} = 10.5 \pm 3.0$ [6], the value $|g| g_{D^*D\pi} = 14.7 \pm 4.2$ is compatible with the experimental data $g_{D^*D\pi} = 17.9 \pm 0.3 \pm 1.9$ [10].

Although the value of the effective mass m_{eff} suffers from large uncertainty, we can expect the final-state interactions improve the value of the strong coupling constant $g_{D^*D\pi}$ significantly. Furthermore, the values of the strong coupling constants $g_{D^*D^*P}$, g_{D^*DP}, f_{D^*DV}, $f_{D^*D^*V}$, g_{DDV}, $g_{D^*D^*V}$ and $g_{\Delta N\pi}$ from the light-cone QCD sum rules are much smaller than most of the existing estimations or experimental data [21, 22]. That maybe a general feature of the light-cone QCD sum rules. We

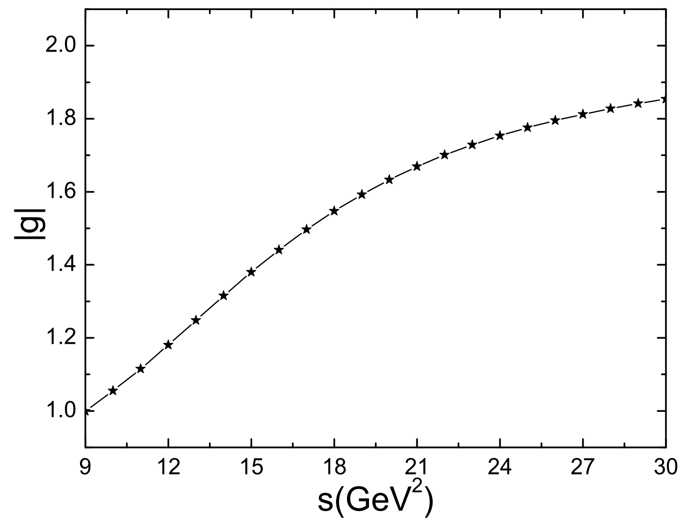

Figure 1: $|g| = |1 - G(s)T_1(s)|$ with s.

perform the operator product expansion at large momentum transfers, experimentally, the momentum transfers in the strong decays do not always warrant validity of the operator product expansion in the light-cone. If we take an assumption that the momentum transfers are large enough, we should take into account all the quantum effects, because at that energy scale, the final-states are active, their interactions may play an important role.

4 Conclusion

In this article, we study the final-state interactions in the strong decay $D^{*+} \to D^0\pi^+$. We take the momentum transfer $D^* \to \pi$ be large to validate the operator product expansion in the light-cone QCD sum rules. At large momentum transfers, the final-state interactions play an important role, and we should take them into account. Although the value of the effective mass m_{eff} of the vector meson D^* suffers from large uncertainty, we can expect the final-state interactions improve the value significantly .

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